In this problem, we'll help the health-conscious consumer use optimization to

determine which and how much of each food to take every day.

Problem 1 - The Decision Variables

How many decision variables will there be in your optimization formulation?

We see there are 32 different foods; we will want to know how much of each to consume.

Problem 2 - The Decision Variables

What type of decision variables should be used in the optimization formulation?

Answer: Continuous variables limited to non-negative values

Problem 3 - Constraints

The consumer needs to satisfy the lower bound and upper bound constraints for the other nutritional requirements (energy, protein, etc.). How many lower bound constraints are there?

Answer: From the table given in the problem we see this is 4.

How many constraints correspond to the upper bound requirements?

Answer: From the table given in the problem we see this is 3.

Problem 4 - Solving the Model

Solve this in Excel

Which foods are selected by the model (non-zero amount)?

Answer: beans, butter, kale, rice

What is the total amount of sugar intake?

Answer: 0.1140696041

Problem 5 - Robustness to Protein Restrictions

The consumer wants to build muscle mass. She reads from her sports magazine that athletes who regularly engage in high-intensity workouts can benefit from more protein intake. The new plan is to change the upper bound of protein from 70 grams to 150 grams.

What's the new total sugar intake?

Answer: 0

Problem 6 - Robustness to Food Availability

(From now on, revert back to the protein upper limit of 70 grams.)

In many parts of the world, the availability of foods, especially fresh fruits and vegetables, can be somewhat limited. Suppose the customer lives at a place where there is only access to rice, beans, cabbage, and wheat flour. Which ones are selected by the model?

Answer: We add the restrictions that all decision variables are equal to 0 except those corresponding to rice, beans, cabbage and wheat flour and find that rice, beans and cabbage are selected by the model.

What's the new total sugar intake?

2.32212

Problem 7 - Robustness to new Constraints

(From now on, revert the food availability to all items.)

Suppose we want to add extra constraints in other nutritional components, such as calcium, Vitamin A, etc.

What could happen to optimal sugar amount? Select all that apply.

Answer: Because this is a minimization problem, adding more constraints will either increase the optimal sugar amount or leave it unchanged.

Problem 8 - But I Don't Like Rice

The consumer understands this ultra-low sugar diet is probably good, but just can't make herself eat rice and beans all the time. She decides to use her preference measure, the "happiness" column in the data where a 1-5 rating is given to each food (with 5 giving her most happiness). She now wants to maximize the total happiness, of course still satisfying all the constraints, with an additional constraint:

sugar upper limit of 30 grams.

Which items are recommended this time?

Answer: asparagus, butter, carrots, cereal

Is the optimal sugar amount at the limit of 30 grams, or below?

Answer: at limit

Problem 9 - Sensitivity to Happiness

The consumer decides that a rating of "3" for carrots is too high, especially if she eats a lot of them. She wants to downgrade it to a "2" and see what changes to her diet. She is still maximizing her happiness, with sugar limited to 30 grams.

Instead of carrots, which one is selected now?

Answer: cookies